MATHEMATICAL DESCRIPTION AND CALCULATION OF CONTACT MELTING

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Heat- and mass-transfer processes are investigated in the contact melting of solids with large specific loads and energy rates appropriate to conditions of thermal drilling.

Many technological processes are based on contact melting of solids according to the scheme in Fig. 1. In the working part of the heating device 1, heat sources 2 are distributed. The temperature t_H at the heating (working) surface Σ_H of the device is above the melting point t_M of the solid mass 4 filling an infinite region Ω of three-dimensional space with moving sections of the boundary — the melting surface Σ_M and the boundaries forming here, the surface of the molten zone Σ_1. The heating device moves into the molten solid medium at some velocity V; the melt formed 3 is squeezed out from the gap between surfaces Σ_H and Σ_M under the action of external load W (the weight force) applied to the system; the load is calculated per unit area of the middle of the working-region cross section generating the thermal energy.

Melting occurs in this way in contact fusion apparatus [1], which is widely used in industry, and in the fusional drilling of rock and glaciers [2-4]; an analogous picture appears in arc welding (fusional welding) [5, 6] and in a whole series of other fields. Although it is widespread, this complex multiparameter process has been the subject of little theoretical study. The development of the corresponding problems of mathematical modeling is especially critical in solving the technical problems arising in the course of new developments. The latter relates directly to a new method of drilling rocks and glaciers — thermal drilling.

Consider a steady process of contact melting. In this case, preliminary investigations [7] of the general equations of mass, momentum, and energy conservation of a molten layer using methods of similarity theory and dimensionality analysis allow the following dimensionless complexes characterizing the heat and mass transfer in the liquid and solid phases to be determined:

$$\mathrm{Pe}=\frac{Vl}{a_S},\ \mathrm{K}_\lambda=\frac{\lambda_S}{\lambda_L},\ \mathrm{K}_c=\frac{c_S}{c_L},\ \mathrm{K_M}=\frac{L}{c_L(t_M-t_\infty)},\ \mathrm{K}_\varkappa=\varkappa l,$$
$$\mathrm{K}_h=\left(\frac{\rho_S\mu_M V}{\rho_L l W}\right)^{1/3},\ \mathrm{K}_g=\frac{\rho_L g l}{W},\ \mathrm{Re}=\frac{Vl\rho_S}{\mu_M}\mathrm{K}_h, \tag{1}$$
$$\mathrm{Br}=\frac{W}{c_L\rho_L(t_M-t_\infty)}\mathrm{Pe_L K}_h\left(\mathrm{Pe_L}=\frac{\mathrm{Pe K}_\lambda}{\mathrm{K}_c},\ \mathrm{K}_W=\frac{\mathrm{K}_h}{\mathrm{Pe}_L^{1/3}}\right).$$

Each of the similarity complexes in Eq. (1) has a completely determined and obvious physical meaning, so that comparison of the orders of magnitude of these quantities for any specific class of processes allows their basic features to be elucidated and permits well-founded simplification of the general mathematical model. Thus, for example, for thermal drilling of rocks and glacier masses, relatively large specific loads W ~ 100 kN/m² are typical, as well as heat fluxes at the working surface and the melting surface q_H, q_M ~ 300 kW/m². As a result V ~ 10^{-4}–10^{-3} m/sec, and when l ~ $5\cdot10^{-2}$ m, Peclet numbers Pe, $\mathrm{Pe_L}$ ~ 10–100; K_λ, K_c, K_M ~ 1. At the same time, K_h, the ratio of the characteristic thickness of the molten layer to the characteristic dimension of the working part of the heating device (the but of the thermodrill) l, is of the order of 10^{-3}–10^{-2}; K_g, the ratio of the scale of the gravitational mass forces in the molten layer to the external load, is 10^{-3}; the Reynolds

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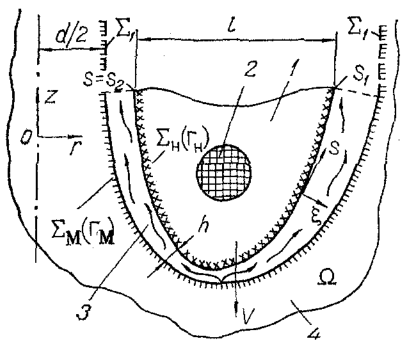

Fig. 1. Diagram of the contact-melting process: 1) heating device; 2) heat-generating element; 3) molten layer; 4) melting mass.

number Re ~ 10^{-7}–10^{-4}; the Brinkman number, determining the intensity of dissipative heating, is Br ~ 10^{-5}–10^{-4}.

The smallness of the liquid-phase layer thickness h ($h \sim K_h l$) allows the boundary-layer approximation to be used in the mathematical description of the heat- and mass-transfer processes occurring there.

Consider the cases of axisymmetric and plane geometry of the contact-melting picture. As shown in Fig. 1, systems of cylindrical (plane Cartesian) coordinates r, z and curvilinear coordinates s, ξ associated with the heating device are introduced. Then, in the region of smooth sections of the generatrix Γ_H of the working surface, its dimensionless curvature is given by the complex $K_\varkappa = O(1)$ and, accurate to quantities of order $O(K_h^2,\ K_\varkappa K_h,\ K_g,\ \mathrm{Re},\ \mathrm{Br})$, the mean (over the molten-layer thickness) dimensionless balance equations of mass, momentum, and energy conservation will take the form

$$R_H^{-\nu}\frac{d}{dS}\left(R_H^{\nu}H\int_0^1 U(S,\ \eta)\,d\eta\right)=\frac{dR_H}{dS}+K_h\frac{dH}{dS}\,\frac{dZ_H}{dS},$$
$$H^2\frac{dP}{dS}=\left.\frac{\partial U}{\partial\eta}\right|_{\eta=1}-\bar{\mu}\left.\frac{\partial U}{\partial\eta}\right|_{\eta=0},\tag{2}$$
$$R_H^{-\nu}\,\mathrm{Pe_L}\frac{d}{dS}\left(R_H^{\nu}H\int_0^1 U(S,\ \eta)\,\Theta_L(S,\ \eta)\,d\eta\right)=Q_H-Q_M.$$

Here

$$(R,\ Z,\ S)=(r,\ z,\ s)/l;\ \eta=\xi/h,\ \bar{\mu}=\mu_H/\mu_M;\ P=p/W;$$
$$H=\frac{h}{lK_h};\ U=\frac{u\rho_L}{V\rho_S}K_h;\ \Theta_{H,L}=\frac{t_{H,L}-t_M}{t_M-t_\infty};\ Q_{H,M}=\frac{lq_{H,M}}{\lambda_L\,(t_M-t_\infty)};\tag{3}$$

$R = R_H(S)$, $Z = Z_H(S)$ are normal equations of the curve Γ_H, $S_2 \leqslant S \leqslant S_1$.

Note that, in calculating the melting of a solid medium by a piecewise-smooth heating surface Σ_H, Eq. (2) requires the addition of special matching conditions, which are valid in the vicinity of the node points of the genetratix Γ_H where $K_\varkappa \to \infty$.

It may be shown that the heat-flux density Q_H at Σ_H is found, accurate to small terms of order $O(K'_\lambda K_h)$, where $K'_\lambda = \lambda_H/\lambda_L$, as a result of solving the individual problem of the temperature conditions of the heating device, when $\Theta_H = 0$ is specified approximately at the working surface. In connection with this, the distribution of Q_H is regarded as known below.

The expression for the heat-flux density Q_M at the melting surface Σ_M is determined by a relation of Stefan-condition type

$$Q_M = K_\lambda Q + Pe_L K_M \left(\frac{dR_н}{dS} + K_h \frac{dZ_н}{dS} \frac{dH}{dS} \right), \tag{4}$$

in which $Q = q_S l / \lambda_S (t_M - t_\infty)$ is the reduced heat-flux density arriving at the solid phase.

The quantity Q is one of the basic characteristics of the contact-melting process, and its calculation is based on the analysis of the problem analogous to the classical problem of welding theory concerning the temperature field $\Theta_S(M)$ in the solid phase around the welding tank [8, 9]

$$\Delta\Theta_S - Pe\, \partial\Theta_S/\partial Z = 0,\ M \in \Omega;$$
$$a)\ \Theta_S|_{\Sigma_M} = 1;\ b)\ \Theta_S|_{\Sigma_1} = \Theta_1;\ c)\ \lim_{M\to\infty} \Theta_S = 0. \tag{5}$$

Here Θ_1 is the temperature distribution at the surface Σ_1 of the molten region (Fig. 1); $\Theta_S = (t_S - t_\infty)/(t_M - t_\infty)$.

In Eq. (5), it is expedient to use the method of boundary integral equations [10], which allows an integral equation of the first kind in Q to be obtained directly without the laborious procedure of constructing its general solution. In the general case, this equation has a fairly complex form and includes integral terms describing the influence of the boundary conditions (temperature field $_1$) at the surface Σ_1 on the heat-flux density Q reaching the solid phase from the melting surface Σ_M. Special estimates show that, in the given range $Pe \geqslant 10$, the distribution of Q at Σ_M is practically independent of the character of the heat transfer at Σ_1: the corresponding perturbations are localized in a small vicinity of the matching line of the surfaces Σ_1 and Σ_M of width $O(Pe^{-1})$, and their integral contribution to the overall heat balance is of order Pe^{-2}. As a result, there is no need to specify the distribution of the temperature Θ_1 at Σ_1; the integral equation for Q is significantly simplified and takes the form

$$\int_{\Gamma_M} Q(M) \exp\left[\frac{Pe}{2}(Z_0 - Z)\right] \varphi(M, M_0)\, d\Gamma = 1,\ M_0 \in \Gamma_M, \tag{6}$$

where Γ_M is the generatrix of the melting surface, and the core $\varphi(M, M_0)$ is determined by the relations: a) in a Cartesian coordinate system in the plane

$$\varphi(M, M_0) = \frac{1}{2\pi} K_0 \left(\frac{Pe}{2} \sqrt{(R - R_0)^2 + (Z - Z_0)^2} \right), \tag{7}$$

where $K_0(x)$ is a zero-order modified Bessel function of the second kind; b) in the presence of axial symmetry in a cylindrical coordinate system

$$\varphi(M, M_0) = \frac{R}{\pi} \int_\alpha^\beta \frac{\exp(-Pe\, t/2)\, dt}{\sqrt{(\beta^2 - t^2)(t^2 - \alpha^2)}}, \tag{8}$$
$$\alpha(M, M_0) = \sqrt{(R - R_0)^2 + (Z - Z_0)^2},\ \beta(M, M_0) = \sqrt{(R + R_0)^2 + (Z - Z_0)^2}.$$

Note that, in thermal-drilling practice, thermodrilling equipment with annular bits intended for taking a sample (a core) of the molten rock is widely used. The symmetry axis Z does not intersect the generatrix Γ_M of the melting surface in this case, and in Eqs. (6) and (8) the difference $\beta - \alpha > D$, the ratio of the core diameter d to the bit width l. Then, when $PeD \gg 1$, a uniformly applicable asymptotic expansion may be constructed for the core in Eq. (8) [11]; the principal term of this expansion differs from Eq. (7) by the factor $\sqrt{R/R_0}$, and the next term is of order $O(Pe^{-1}D^{-1})$. The result obtained significantly simplifies the calculation of contact-melting processes due to annular heating elements and illustrates the physically obvious equivalence of the formulation of the corresponding mathematical problems in plane Cartesian and cylindrical coordinate systems as $PeD \to \infty$.

Equations (2), (4), and (6) may provide the basis for constructing closed mathematical models of contact-melting processes for large specific loads ($K_h \ll 1$) and rates of melting ($Pe \gg 1$).

The quadratic polynomial approximations with respect to η of the temperature profiles and longitudinal velocity component of the melt flow in the gap between the working surface of the heating instrument and the melting surface are now specified, and the mean delivery temperature $\bar{\Theta}(S)$ and longitudinal velocity $\bar{U}(S)$ of the liquid phase are introduced. Taking account of the obvious boundary conditions at Σ_H ($\eta = 0$) and Σ_M ($\eta = 1$), and substituting the resulting expressions for $\Theta(S, \eta)$ and $U(S, \eta)$ into Eq. (2) with relative error $O(K_h)$, it is found that

$$\bar{U}(S) = \frac{R_H^{\gamma+1} - R_*^{\gamma+1}}{(\gamma+1) H R_H^{\gamma}}, \quad P(S) = P_1 + \frac{6}{\gamma+1} \int\limits_{S}^{S_1} \frac{(1+\bar{\mu})(R_H^{\gamma+1} - R_*^{\gamma+1})}{H^3 R_H^{\gamma}} dS. \tag{9}$$

It is assumed here that in the region of the ends of the working-surface generatrix ($S = S_1$, $S = S_2$) specified pressure values are maintained: $P(S_1) = P_1$ and $P(S_2) = P_2$. The second of these conditions allows the constant of integration R_* in Eq. (9) to be found:

$$R_*^{\gamma+1} = \left(P_1 - P_2 + \int\limits_{S_2}^{S_1} \frac{R_H dS}{H^3}\right) \Big/ \int\limits_{S_2}^{S_1} \frac{dS}{H^3 R_H^{\gamma}},$$

which, if $R_* \in [R_H(S_2), R_H(S_1)]$, may be interpreted as the coordinate of the branch point of the melt flow, where $\bar{U} = 0$.

Note that in the case where the heating instrument is a solid of revolution, and the axis Z (the axis of revolution) intersects the surface Σ_H, the condition $P(S_2) = P_2$ is meaningless; in view of the axial symmetry assumed for the melting process, $R_* = R_H(S_*) = 0$, $S_* = S_2$, and it is expedient to set $S_2 = 0$.

Further, in a quadratic approximation in η, the temperature profile in the melt layer is

$$Q_H = (7\Theta_H - 10\bar{\Theta})/(2K_h H), \tag{10}$$

and energy equation (2) and Eq. (4) take the following form:

$$\begin{gathered} \mathrm{Pe_L} \frac{d}{dS} [(R_H^{\gamma+1} - R_*^{\gamma+1}) \bar{\Theta}] = \frac{5(\gamma+1) R_H^{\gamma}(\Theta_H - 2\bar{\Theta})}{K_h H}, \\ K_M \mathrm{Pe_L} \left[K_h \frac{dZ_H}{dS} \frac{dH}{dS} + \frac{dR_H}{dS}\right] + K_\lambda Q = \frac{10\bar{\Theta} - 3\Theta_H}{2K_h H}. \end{gathered} \tag{11}$$

The first and second relations in Eq. (11) are degenerate at the points of S where $R_H(S) = R_*(S = S_*)$ and $dZ_H/dS = 0$, respectively. It is natural to require that the derivatives $d\bar{\Theta}/dS$ and dH/dS be bounded here; this gives the boundary conditions necessary for solving Eq. (11). In addition, it is obvious that the axial load applied from the direction of the heating-instrument working surface to the melt layer is balanced by the internal-stress forces in the liquid phase, which, accurate to terms of order $O(K_h)$, are the pressure forces. Thus, taking account of Eqs. (3) and (9), it is not difficult to write the additional condition

$$\frac{6}{\gamma+1} \int\limits_{S_2}^{S_1} \frac{(1+\bar{\mu})(R_H^{\gamma+1} - R_*^{\gamma+1}) R_H^{2-\gamma}}{H^3} dS + (P_1 - 1) R_H^2(S_1) - (P_2 - 1) R_H^2(S_2) = 0. \tag{12}$$

The system of Eqs. (6)-(8), (10)-(12) is closed and, if the distribution $Q_H(S)$ is specified, allows characteristics of the contact-melting process such as $Q(S)$, $\Theta_H(S)$, $\bar{\Theta}(S)$, $H(S)$, and Pe to be found, and then $\bar{U}(S)$ and $P(S)$ are calculated from Eq. (9).

The generatrix of the melting surface Γ_M may be specified under the condition of smoothness of Γ_H by the parametric equations

$$\begin{gathered} R = R_M(S) \equiv R_H(S) + K_h H(S)\, dZ_H/dS, \\ Z = Z_M(S) \equiv Z_H(S) - K_h H(S)\, dR_H/dS. \end{gathered}$$

Hence, in the case of stability of the solution $Q(S)$ of Eq. (6) with respect to small perturbations of the melting surface, it is possible, in the first approximation, with an expected error of order $O(K_h)$, to replace the integration curve Γ_M in the integral in Eq. (6) by Γ_H and

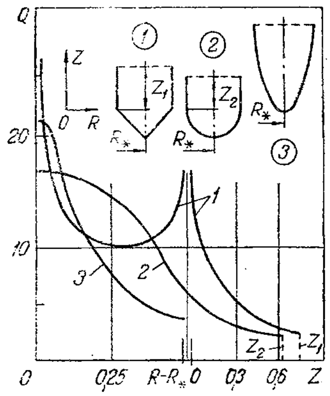

Fig. 2. Distribution of heat-flux density reaching the solid body from the melting surface, for the case of an annular heating instrument with Pe = 15. The form of Σ_M is conical (1), spherical (2), and parabolic (3).

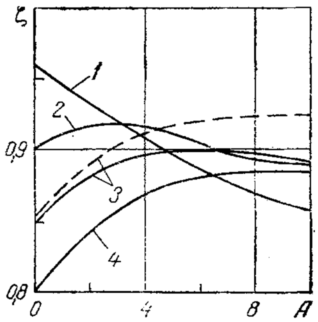

Fig. 3

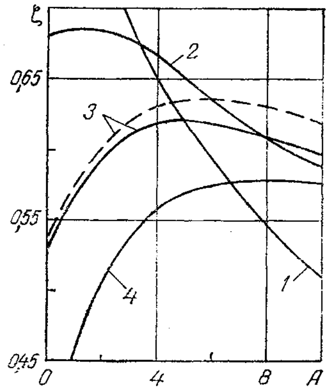

Fig. 4

Fig. 3. Dependence of the efficiency ζ of a parabolic annular thermodrilling bit on the elongation parameter A in the fusional drilling of ice; $K_W = 1.4 \cdot 10^{-4}$; $K_\lambda = 4.1$; $K_c = 0.5$; Pe = 15 (1), 30 (2), 45 (3), 60 (4); $K_M = 1.6$ (solid curves), 4 (dashed curve).

Fig. 4. Dependence of the efficiency ζ of a paraboloidal thermodrill bit with a continuous face on the elongation parameter A in the fusional melting of rock: $K_W = 0.77 \cdot 10^{-2}$; $K_\lambda = 1.06$; $K_c = 1.1$; Pe = 25 (1), 50 (2), 75 (3), 100 (4); $K_M = 0.16$ (solid curves), 0.3 (dashed curve).

realize an iterative algorithm for the construction of the solution of Eqs. (6) and (10)-(12). As a result, the problem of investigating and solving Eq. (6) takes on independent importance.

In the practically important particular case when the genetratix of the melting surface Γ_M is a parabola $Z_M = A(R_M - R_*)^2$ with an elongation parameter A > 0, the results of analyzing Eq. (5) [11, 12] may be used, and the solution of Eq. (6) written in the form

$$Q = \frac{\mathrm{Pe}\exp(-\alpha^2)}{\sqrt{1+4AZ_M}} \Bigg/ \left\{ \begin{matrix} \sqrt{\pi}\alpha\,\mathrm{erfc}(\alpha), & \gamma = 0 \\ \alpha^2\,\mathrm{Ei}(-\alpha^2), & \gamma = 1 \end{matrix} \right\}, \tag{13}$$

where

$$\alpha^2 = \mathrm{Pe}/(4A);$$

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-u^2)\,du, \quad \mathrm{Ei}(-x) = \int_1^\infty \frac{\exp(-xu)}{u}\,du.$$

An algorithm for the numerical solution of Eq. (6) on the basis of the Krylov–Bogolyubov method has been developed for the calculation of contact-melting processes with heating-instrument working surfaces of arbitrary form. The results of calculating the distribution of Q for some types of Γ_M are shown in Fig. 2. It is readily evident that the presence of node points on the generatrix of the working surface of the heating instrument Γ_H will lead to an anomalous increase in heat-flux density Q in their vicinity at small K_h, to unreasonable heating of the melt layer in the intermediate regions and, finally, to reduction in efficiency of the heating instrument.

Within the framework of the given mathematical model, in view of Eq. (10), specifying the heat-flux density Q_H on Σ_H is equivalent to specifying the distribution of the temperature Θ_H. In particular, the case of a heater with an isothermal working surface, which is often encountered in thermal-drilling practice, for example, may be studied in this way. If in addition the condition $Q/(dR_H/dS) = B = \text{const}$ is satisfied here, at least approximately, as for a bit of parabolic form — see Eq. (13) — then the solution of Eqs. (10)-(12) is written in the explicit form

$$\bar{H}^3 = \frac{6(1+\bar{\mu})}{(\gamma+1)[(1-P_1)R_{\text{н}}^2(S_1)-(1-P_2)R_{\text{н}}^2(S_2)]}\int\limits_{S_1}^{S_2}(R_{\text{н}}^{\gamma+1}-R_*^{\gamma+1})R_{\text{н}}^{2-\gamma}\left(\frac{dR_{\text{н}}}{dS}\right)^3 dS,$$

$$\begin{aligned}\bar{Q}_{\text{н}} &= \mathrm{Pe}_L\bar{B}(1+0.35\,\mathrm{Pe}_L\,\mathrm{K}_h\bar{H})/(1-0.15\,\mathrm{Pe}_L\,\mathrm{K}_h\bar{H}),\\ \bar{\Theta} &= 0.5\,\mathrm{Pe}_L\,\mathrm{K}_h\bar{B}\bar{H}/(1-0.15\,\mathrm{Pe}_L\,\mathrm{K}_h\bar{H}),\\ \Theta_{\text{н}} &= \mathrm{Pe}_L\,\mathrm{K}_h\bar{B}\bar{H}(1+0.1\,\mathrm{Pe}_L\mathrm{K}_h\bar{H})/(1-0.15\,\mathrm{Pe}_L\,\mathrm{K}_h\bar{H}).\end{aligned} \tag{14}$$

Here $\bar{H} = HdR_{\text{н}}/dS$, $\bar{Q}_{\text{н}} = Q_{\text{н}}/(dR_{\text{н}}/dS)$, $\bar{B} = \mathrm{K_M} + B\,\mathrm{K}_\lambda/\mathrm{Pe}_L$.

Equation (14) is suitable for rough engineering calculations.

To compare different thermodrilling-equipment bits (penetrators), the efficiency ζ is introduced as a measure of the effectiveness of their form and construction. It is defined as the ratio of the minimum thermal-energy supply rate N_0 necessary for melting the rock in the middle cross section of the bit to the power N_H taken from the working surface:

$$\zeta = \frac{N_0}{N_{\text{н}}},\quad N_{\text{н}} = 2\pi\int\limits_{S_2}^{S_1} R_{\text{н}}Q_{\text{н}}dS,$$

$$N_0 = \pi[R_{\text{н}}^2(S_1) - R_{\text{н}}^2(S_2)](\mathrm{K_c} + K_M)\,\mathrm{Pe_L}.$$

Calculations based on the mathematical model of contact melting outlined in Eqs. (6)-(12) with a fixed volume of a heating instrument with an isothermal working surface show that, other conditions being equal, ζ is closest to a maximum in parabolic heaters.

The efficiency ζ is shown as a function of the elongation parameter A in Figs. 3 and 4 for fusional drilling of glaciers and rocks by means of thermodrilling equipment with annular parabolic bits (with an annular face) and with bits in the form of paraboloids of revolution (continuous face). Analysis of these curves shows that at small A, heat losses on account of nonsteady heating of the liquid phase predominate; at large A, energy losses associated with heat scattering in the molten mass become more considerable. The balance of these two components depends on the rate of melting, and the maximum of ζ is shifted toward larger A with increase in Pe.

NOTATION

a, thermal diffusivity; c, specific heat; d, core diameter; g, acceleration due to gravity; h, thickness of molten layer; l, characteristic dimension of heating device (width or diameter); L, latent heat of fusion; M, M_0, points of space; p, pressure; q, heat-flux density; r, z, Cartesian or cylindrical coordinates, defined in Fig. 1; s, ξ, longitudinal and transverse coordinates in the molten layer, defined in Fig. 1; S_2, S_1, coordinates of the end points of the heating-surface generatrix; t, temperature; u, longitudinal velocity in the molten layer; V, melting rate; W, specific axial load from the direction of the heating instrument; α, β, limits of integration in Eq. (8); γ, parameter equal to 0 for a plane pattern of the melting process and to 1 for an axisymmetric pattern; Γ, generatrix of the surface Σ; Δ, Laplacian operator; ζ, efficiency; $\varkappa$, characteristic value of the heating-surface curvature; λ, thermal conductivity; μ, dynamic viscosity; ρ, density; Σ, surface; $\varphi(M, M_0)$, integral core in Eq. (6). Dimensionless numbers, parameters, and functions: Br, K_c, K_g, K_h, K_W, K_λ, $K_\varkappa$, K_M, Pe, Pe_L, Re, defined in Eq. (1); H, P, Q_H, Q_M, R, S, U, η, $\bar{\mu}$, Θ_H, Θ_L, Q, defined in Eqs. (3) and (4); Θ_S, Θ_1, defined in Eq. (5). Indices: L, liquid phase; H, heating-element surface; S, solid phase; M, melting surface; *, branch point of flux; ∞, value at infinitely remote points.

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